Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MPC4

Unit Pure Core 4

Tuesday 9 June 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 It is given that $\mathrm{f}(x) = \dfrac{19x - 2}{(5 - x)(1 + 6x)}$ can be expressed as $\dfrac{A}{5 - x} + \dfrac{B}{1 + 6x}$, where A and B are integers.

(a) Find the values of A and B.

[3 marks]

(b) Hence show that $\displaystyle\int_0^4 \mathrm{f}(x)\,\mathrm{d}x = k \ln 5$, where k is a rational number.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE	Answer space for question 1

2 (a) Express $2\cos x - 5\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, giving your value of α, in radians, to three significant figures.

[3 marks]

(b) (i) Hence find the value of x in the interval $0 < x < 2\pi$ for which $2\cos x - 5\sin x$ has its maximum value. Give your value of x to three significant figures.

[2 marks]

(ii) Use your answer to part **(a)** to solve the equation $2\cos x - 5\sin x + 1 = 0$ in the interval $0 < x < 2\pi$, giving your solutions to three significant figures.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE	**Answer space for question 2**

3 (a) The polynomial $f(x)$ is defined by $f(x) = 8x^3 - 12x^2 - 2x + d$, where d is a constant. When $f(x)$ is divided by $(2x + 1)$, the remainder is -2. Use the Remainder Theorem to find the value of d.

[2 marks]

(b) The polynomial $g(x)$ is defined by $g(x) = 8x^3 - 12x^2 - 2x + 3$.

(i) Given that $x = -\frac{1}{2}$ is a solution of the equation $g(x) = 0$, write $g(x)$ as a product of three linear factors.

[3 marks]

(ii) The function h is defined by $h(x) = \dfrac{4x^2 - 1}{g(x)}$ for $x > 2$.

Simplify $h(x)$, and hence show that h is a decreasing function.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE	Answer space for question 3

4 (a) Find the binomial expansion of $(1+5x)^{\frac{1}{5}}$ up to and including the term in x^2.

[2 marks]

(b) (i) Find the binomial expansion of $(8+3x)^{-\frac{2}{3}}$ up to and including the term in x^2.

[3 marks]

(ii) Use your expansion from part **(b)(i)** to find an estimate for $\sqrt[3]{\frac{1}{81}}$, giving your answer to four decimal places.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 4

QUESTION PART REFERENCE

Answer space for question 4

Turn over ▶

5 A curve is defined by the parametric equations $x = \cos 2t$, $y = \sin t$.

The point P on the curve is where $t = \dfrac{\pi}{6}$.

(a) Find the gradient at P.

[3 marks]

(b) Find the equation of the normal to the curve at P in the form $y = mx + c$.

[3 marks]

(c) The normal at P intersects the curve again at the point $Q(\cos 2q,\ \sin q)$.

Use the equation of the normal to form a quadratic equation in $\sin q$ and hence find the x-coordinate of Q.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE

Answer space for question 5

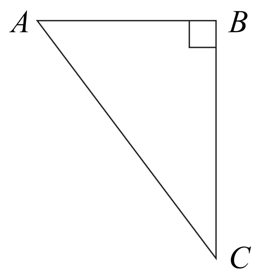
6 The points A and B have coordinates $(3,\ 2,\ 10)$ and $(5,\ -2,\ 4)$ respectively.

The line l passes through A and has equation $\mathbf{r} = \begin{bmatrix} 3 \\ 2 \\ 10 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$.

(a) Find the acute angle between l and the line AB.

[4 marks]

(b) The point C lies on l such that angle ABC is $90°$.



Find the coordinates of C.

[4 marks]

(c) The point D is such that BD is parallel to AC and angle BCD is $90°$. The point E lies on the line through B and D and is such that the length of DE is half that of AC.

Find the coordinates of the two possible positions of E.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 6

QUESTION PART REFERENCE	Answer space for question 6

Turn over ▶

QUESTION PART REFERENCE	Answer space for question 6

QUESTION PART REFERENCE	Answer space for question 6

7 A curve has equation $y^3 + 2e^{-3x}y - x = k$, where k is a constant.

The point $P\left(\ln 2, \frac{1}{2}\right)$ lies on this curve.

(a) Show that the exact value of k is $q - \ln 2$, where q is a rational number.

[1 mark]

(b) Find the gradient of the curve at P.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 7

QUESTION PART REFERENCE	Answer space for question 7

8 (a) A pond is initially empty and is then filled gradually with water. After t minutes, the depth of the water, x metres, satisfies the differential equation

$$\frac{dx}{dt} = \frac{\sqrt{4+5x}}{5(1+t)^2}$$

Solve this differential equation to find x in terms of t.

[7 marks]

(b) Another pond is gradually filling with water. After t minutes, the surface of the water forms a circle of radius r metres. The rate of change of the radius is inversely proportional to the area of the surface of the water.

(i) Write down a differential equation, in the variables r and t and a constant of proportionality, which represents how the radius of the surface of the water is changing with time.

(You are not required to solve your differential equation.)

[3 marks]

(ii) When the radius of the pond is 1 metre, the radius is increasing at a rate of 4.5 metres per second. Find the radius of the pond when the radius is increasing at a rate of 0.5 metres per second.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 8

QUESTION PART REFERENCE	Answer space for question 8

Turn over ▶

QUESTION PART REFERENCE

Answer space for question 8

END OF QUESTIONS